Bolsa de mestrado: "Origins of generic points in algebraic geometry and mathematical practice"

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Abstract

In the last century, the abstract trend of pure mathematics deeply influenced our perspective of what mathematics is about. The difficulty is keeping with a sufficiently solid notion of intuition, able to sanction the development of mathematics, had the effect on the one hand of developing logical tools, whose application does not depend on the content of our reasoning, while on the other hand to develop purely mathematical notions able to transcend the limit of our power of expressivity.

Specifically, an important notion that seemed to evolve in algebraic geometry and in functional analysis, in the last decades, is that of generic point: points that represent an idealisation with respect to other points that belong to the same mathematical structure. Such points are seen as having all properties that analog points have in their context of existence, but they cannot be identified with any of the concrete points whose properties refer to. Their existence, therefore, is intended to show that there are mathematical objects that lay beyond our possibility of mathematical specification.

The plan of this master project consists in giving a rational treatment of the use of arbitrary points in mathematical practice, connecting the definition of generic point that we find in set theory, algebraic geometry and functional analysis.

Description of aims

The aim of this grant is to give to a students the conceptual and technical tools necessary to successfully conclude a master in logic at the philosophy department and to be able to start in the best conditions a PhD thesis on more advanced themes of mathematical logic.

The structure of the proposed research aims at developing the basis of a possibly interesting area of application between different areas of mathematics. Indeed, in connecting the definition of generic points in set theory, algebraic geometry and functional analysis, the project aims at developing the appropriate mathematical connections that can give rise, later, to interesting problems of application of logical tools to different fields of research.

Parallel to the mathematical investigation proposed the aim is to develop a sufficiently deep understanding of logic, able to allow to understand the formal application of the mathematical analogy discovered in the more mathematical part of this project. To this aim the courses offered at the philosophy department at Unicamp will be able to cover all basic areas of mathematical logic (set theory, computability theory, model theory and proof theory).

Specifically, the more abstract aim of this project is to isolate a common trait of all definitions of generic points. Moreover, parallel to this technical work the candidate will also develop an historical inquire into the origin of the term 'generic point' in algebraic geometry and functional analysis.

Timetable with methodology and expected results

The two years of this grant will be divided along the axe historical-technical. The first year will be devoted to form a repertory of the principal definitions of generic point that it is possible to find in the literature. A very good point of departure will be offered by , [Hartshorne, 1997], [Griffiths and Harris, 1994] and [Kunen, 1980].

The second year will be devoted to the study of the connections between the definitions found during the first year. Particular attention will be given to works that already connect set theory and other branches of mathematics, as in [Viale, 2016], and a preliminary very abstract study of forcing, as discussed in [Bowen, 1974]. Besides that it will also be inquired the connection between the use of the Baire's category theorem in functional analysis and the role that Forcing Axioms have in set theory.

Besides the study described above the candidate will attend the course offered at the philosophy department at Unicamp on more technical subjects; in order to develop a logical competence. For what concerns methodology the analysis of texts will be the preferred one, on the historical side.

The result of this master grant will be the writing of a master thesis, bearing the title of the project, from which it is planned to extract at least an article summarising the results of the analysis proposed: a repository of mathematical definitions of generic points, together with an analysis of their common aspects. A perfect journal for the publication of the outcome of this work could be the

journal *Studia Logica*.

Justification of the plan

The justification for this grant and for the detailed explanation of its realisation is twofold. On the one hand, while being a work of understanding more than of creation, the outcome of this research aims to fill in a gap in the mathematical community: the production of a catalog of mathematical definition on an important and transversal subject at the intersection between philosophy of mathematics and logic. On the other hand, it will offer an interesting first step towards the development of the Jovem Pesquisador project that I am proposing. Therefore the outcome of this work will provide useful bibliographical information for the development of the project *Arbitrariness and genericity. Or on how to speak of the unspeakable* a a conceptual clarification that we hope to apply to the understanding of the notion of genericity.

References

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